EUCLID AND HIS MODERN RIVALS

Euclid and His Modern Rivals. By Charles L. Dodgson, M.A. (London: Macmillan, 1879.)

Elementary Geometry. Books i.—iv., containing the subjects of Euclid's first six books; following the syllabus of geometry prepared by the Geometrical Association. By J. M. Wilson, M.A. Fourth edition. (London: Macmillan, 1878.)

BY a curious chance these two works reached our hands nearly on the same day, and as Mr. Dodgson devotes a great portion of his space (62 pp.) to the consideration of Mr. Wilson's Geometries, we have thought it well to notice the two authors at the same time. As however it is patent from the fact of Mr. Wilson's work having reached a fourth edition, that his method is not unknown to, and, may we add, not unappreciated by, a large section of mathematical teachers, we shall at once pass on to a consideration of Mr. Dodgson's book, only noticing Mr. Wilson's book in connection with the criticisms put forward in "Euclid and His Modern Rivals."

A few words by way of introduction. Mr. Dodgson has been a teacher of geometry at Oxford, we believe, for nearly five-and-twenty years, and during that time has had frequent occasion to examine candidates in that subject. For a great part of the above-stated period things went pretty smoothly, and King Euclid held undisputed sway in the "Schools;" but eleven years ago a troubler of the geometrical Israel came upon the scene, and read a paper before the Mathematical Society, entitled "Euclid as a Text-Book of Elementary Geometry." The agitation thus commenced acquired strength, and at length, in consequence of a correspondence carried on in these columns, the Geometrical Association was formed. A prime mover in this matter was that Mr. Wilson who wrote the paper, and subsequently brought out the geometry cited. Mr. Dodgson is one of the gentlemen opposed to this change, and the moving cause of the present Iliad is the "vindication of Euclid's masterpiece." Another consequence of the agitation is that many have tried their prentice hands on the production of new geometries—"rivals," our author calls them—"forty-five were left in my rooms to-day." Can we wonder then, that, his soul being stirred within him, he should overhaul a selection of them to see what blots he could "spot" in them? He might well have taken for his motto one once familiar to us—

"If there's a hole in a' your coats,
I rede ye tent it;
A chiel's amang ye takin' notes,
An' faith he'll prent it!"

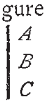
Our author's criticism takes a peculiar form, but we shall not blame him for this, for he has afforded us much amusement, and we quite hold with the Horatian line he cites in extenuation of his mode of procedure: "Ridentem dicere verum quid vetat?" We believe he has made a good many hits, but at times his wit, we think, has led him too far. We shall not, however, here give any account of his plot—we prefer to refer our readers to the work itself—but confine our notice to the remarks upon Mr. Wilson's books, and upon Mr. Morell's "Euclid Simplified."[1]

Mr. Dodgson devotes forty-eight pages to Mr. Wilson's "Elementary Geometry" (second edition, 1869). We can hardly see why so much space should be devoted to a work which seems tacitly to have been withdrawn by the author, or, at any rate, to have been considered inferior to the work under review. Is it that the "scene" was written some time since, and was considered to be too good to be sacrificed? Happily it is not our business to defend Mr. Wilson's views on "direction;" he is perfectly competent to defend his own views, and no doubt, should he see fit, will do so at the right time.

"Minos"—who argues for Mr. Dodgson—himself seems to think that his remarks will now and again be considered hypercritical. Take the following:—

Niemand (the general representative of the "rivals," quoting from the "Elementary Geometry"). Two straight lines that meet one another form an angle at the point where they meet (p. 5). *Min.* Do you mean that they form it "*at* the point," and nowhere else? *Nie.* I suppose so. *Min.* I fear you allow your angle no magnitude, if you limit its existence to so small a locality! *Nie.* Well, we *don't* mean "nowhere else." *Min.* (*meditatively*). You mean *at* the point—and *some*where else? *Where* else, if you please? *Nie.* We mean—we don't quite know why we put in the words at all. Let us say "Two straight lines that meet one another form an angle." *Min.* Very well. It hardly tells us what an angle *is*, and, so far, it is inferior to Euclid's definition; but it may pass. Again (p. 73), *Nie.* reads, P. 5, Ax. 5, "Angles are equal when they could be placed on one another so that their vertices would coincide in position, and their arms in direction." *Min.* "Placed on one another!" Did you ever see the child's game, where a pile of four hands is made on the table, and each player tries to have a hand at the top of the pile? *Nie.* I know the game. *Min.* Well, did you ever see both players succeed at once? *Nie.* No. *Min.* Whenever that feat is achieved you may *then* expect to be able to place two angles "on one another!" You have hardly, I think, grasped the physical fact that, when one of two things is *on* the other, the second is *underneath* the first. But perhaps I am hypercritical.

What the text means is, of course, that *B*, *C*, *D* could be placed upon *A* or *A* upon *B*, *C*, *D*, so as to coincide. A still more striking instance is p. 160. Mr. Wilson adopts the syllabus-definition, "When one straight line stands upon another straight line and makes the adjacent angles equal, each of the angles is called a *right angle*"—a definition, by the way, remarkably like Euclid's. Minos says, "allow me to present you with a figure, as I see the Syllabus does not supply one—



Here *A B* 'stands upon' *B C*, and makes the adjacent angles equal. How do you like these 'right angles?'"

This is a hit, of course, indeed a double hit, the one farcical in its illustration, the other sober enough, for the Syllabus considers that two angles (a major, and a minor, conjugate) are formed by two straight lines drawn from a point. Mr. Dodgson is very amusing upon the "straight" angle, and, no doubt, would be equally so upon the equivalent "flat" angle. A good phrase is still a desideratum, but De Morgan long ago pointed out that "the angle made by a straight line with its continuation is a definite angular magnitude," and considered its half to be the best definition of a right angle.

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[1] A work we ourselves had occasion very strongly to condemn—see NATURE, vol. xiii. p. 202.

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We pass over many passages we had marked, with saying that in many cases the objections are sound but trivial. Objection is taken to Mr. Wilson's remark, "Every theorem may be shown to be a means of indirectly measuring some magnitude," and Niemand abandons "every." We think, however, that Niemand might have made a better fight of it and suggested that what is intended is that, for instance, all the theorems of the first book are directly or indirectly required for the proof of the 47th Proposition, which is surely a proposition concerned with the measurement of magnitude.

On p. 177 Minos says of the exercise, "Show that the angles of an equiangular triangle are equal to two-thirds of a right angle. In this attempt I feel sure I should fail. In early life I was taught to believe them equal to *two right angles*—an antiquated prejudice, no doubt; but it is difficult to eradicate these childish instincts." Mr. Dodgson was taught that the *three* angles were equal to this magnitude; the question says "angles" surely in the plain sense of each angle being equal, &c. Again, in the construction for proposition corresponding to Eucl. i. 9 objection is taken to "finding a radius greater than half AB" (it should be AC): "it would seem to require the previous bisection of AB" (AC). Thus the proof involves the fallacy "*Petitio Principii*." Surely one can take a line greater than or equal to AC; where, then, is the fallacy? Exception is taken to the proposition "the area of a trapezium is equal to the area of a rectangle whose base is half the sum of the two parallel sides, and whose altitude is the perpendicular distance between them" as being "a mere 'fancy' proposition of no practical value whatever." We have met with it in works on co-ordinate geometry and elsewhere. Then again the theorem (Apollonius's) on Mr. Wilson's p. 95 is branded "new," "but even with that mighty name to recommend it, I cannot help thinking it rather more curious than useful." It is our own impression that it is one of the most important "riders" from the second book, and if Mr. Dodgson has been teaching geometry for nearly five-and-twenty years, so have we—but we do not confine our teaching to the text-book only, we devote a great part of our geometrical teaching time to the working of exercises.

Our conclusion from the examination of Mr. Dodgson's objections to Mr. Wilson's last book is that the majority of them can be easily met; indeed, many of them are mere verbal quibbles; the rest arise from the very different standpoints taken up by the two writers, and here there is likely to be "war to the knife."

A word or two on Morell's (J. R.) "Euclid Simplified." It is very easy work to pick this little book to pieces, but we cannot understand a statement of Mr. Dodgson's on p. 148. Of the proposition "Every convex closed line $ABCD$ enveloped by any other closed line $PQRST$ is less than it," he says the method used fails, "as of course all methods must, the thing not being capable of proof." We cannot call to mind any English text-book in which the proposition is proved, but there is what we have thought was a proof in Sannia and D'Ovidio's "Elementi di Geometria," p. 32.

We are bound to say that "Euclid and his Modern Rivals" is not all amusing reading. It alternates

"From grave to gay,"

and more than a third part is devoted to appendices, the third to the sixth of which (73 pages) must have cost the author a great deal of thought and labour. We fear, however, it will not get the attention it deserves. It is hard reading, and one has hardly been led up to it by the amusement provided in the four Acts of the Drama. Some little trouble is involved in mastering the symbols and their significance.

The fourth act considers the objections brought by Mr. Wilson ("Euclid as a Text-Book," &c.) and others against the use of Euclid for junior pupils on the score of unsuggestiveness and want of simplicity of style, the exclusion of hypothetical constructions, &c. We need not consider them here, but refer to two articles by the Rev. Dr. Jones ("On the Unsuitableness of Euclid as a Text-Book of Geometry," *Trans.* of Liverpool Lit. and Phil. Society, published in a separate form; and "Review of Mr. Todhunter's Essay on Elementary Geometry," (*Monthly Journal of Education*, 1875, pp. 97-112, 150-160), neither of which is referred to by our author, though he quotes largely in the appendix from Mr. Todhunter's Essay and also from a review of Mr. Wilson's first Geometry in the *Athenæum* for July 18, 1868, written by Prof. De Morgan. We could instance other geometries which have an equal claim to be considered with any of those criticised by Mr. Dodgson, and we should rather have written "Euclid and *some* of his Modern Rivals."

LETTERS TO THE EDITOR

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The Papau or Papaye

IN NATURE, vol. xix. p. 447, is a paragraph relative to the singular qualities of the *Carica papaya*. I cannot but think that some of the properties attributed to this vegetal in British Guiana by the natives of that colony are exaggerated somewhat, *e.g.*, the tempering of steel by its sap, &c.

Sir Wyville Thomson, in the first volume of "The Voyage of the *Challenger*," gives a capital representation of a group of these papaw-trees in the garden of the Admiral commanding on the North American station at Clarence Hill, Bermudas, where they seem to abound; I do not know if these diœcious plants are indigenous to these islands or introduced from the West Indies and tropical America. From the cut above mentioned can be seen the quaint growth of these paradoxical trees, which must have been esteemed by the early voyagers, as they have been introduced into all parts of the tropics. The singular-looking straight stems (not unlike the gigantesque tree-cabbage stalks of the Channel Islands) are crowned with a tuft of digitate leaves, somewhat at a distance resembling those of the *Aralia papyrifera*, under which the clusters of black purple fruit protrude. In the islands of Bourbon and Mauritius they make a passable *compôte* of these fruit, which are pulpy and full of black seeds when ripe, and the Creole children eat them raw, with what effect on their insides I know not; the birds, however, will not touch them, and as they fall they rot on the ground beneath. In Mauritius, where we lived principally on ration beef cut from the tough flesh of Malagasy oxen, we were in the habit of hanging the ration under the leaves of the tree itself, and if we were in a hurry for a very tender piece of *filet*, our cook would wrap up the undercut of the sirloin in the leaves, when the newly-killed meat would be as tender as if it had been hung for a considerable time. Whence are these deleterious effects causing rapid decomposition of animal fibre? and are there any other trees which possess similar properties?

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